Appendix C
Derivation of the General Wedge Equation

The equations for sliding stability analysis of a general wedge system are based on the right-hand sign convention that is commonly used in engineering mechanics. The origin of the coordinate system for each wedge is located in the lower left-hand corner of the wedge. The x- and y-axes are horizontal and vertical, respectively. Axes that are tangent (t) and normal (n) to the failure plane are oriented at an angle (α) with respect to the +x- and y-axes. A positive value of α is a counter-clockwise rotation; a negative value of α is a clockwise rotation.

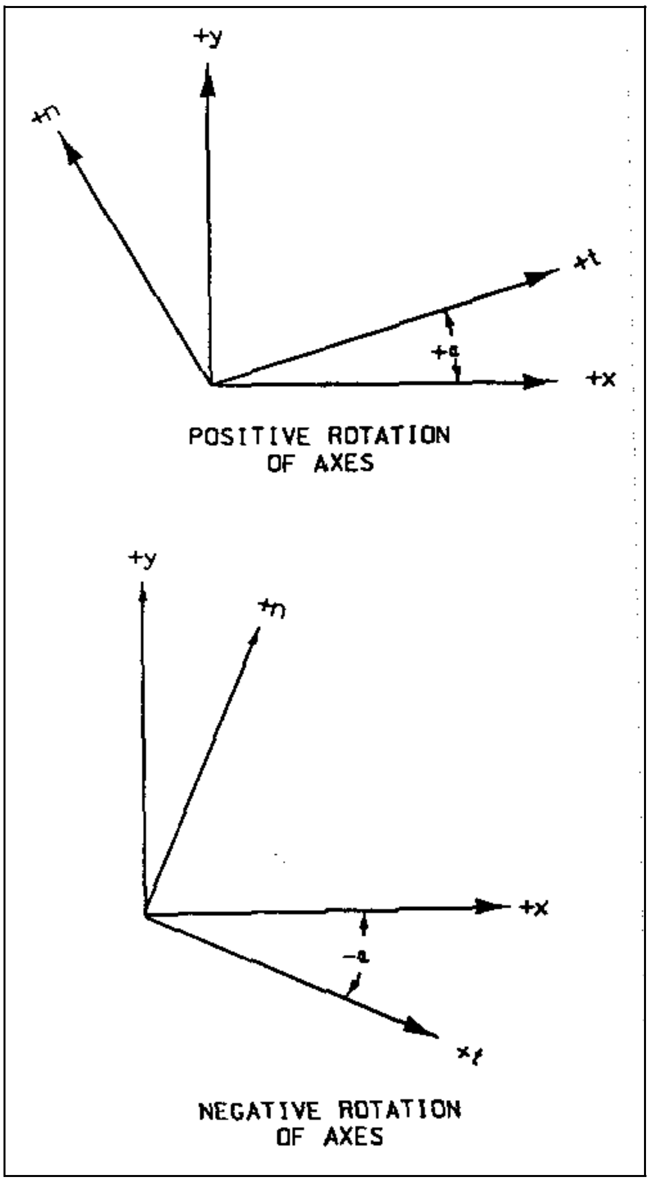


Figure C-1. Sign convention for geometry

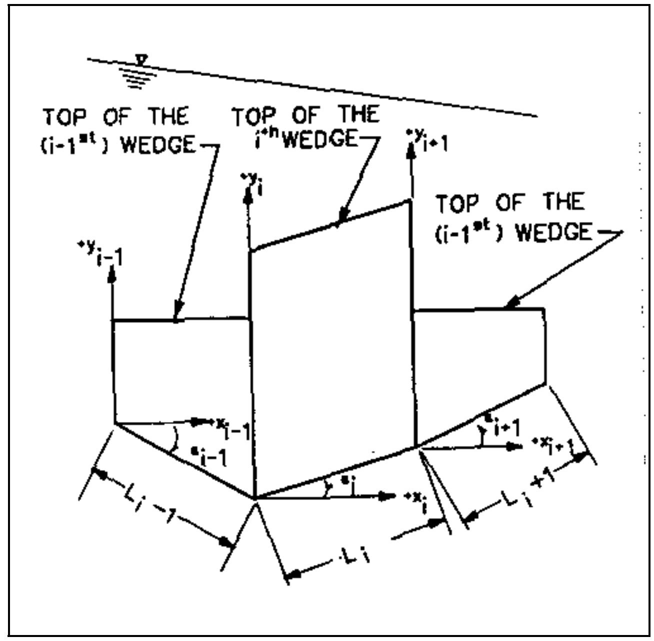


Figure C-2. Geometry of the typical i^{th} wedge and adjacent wedges

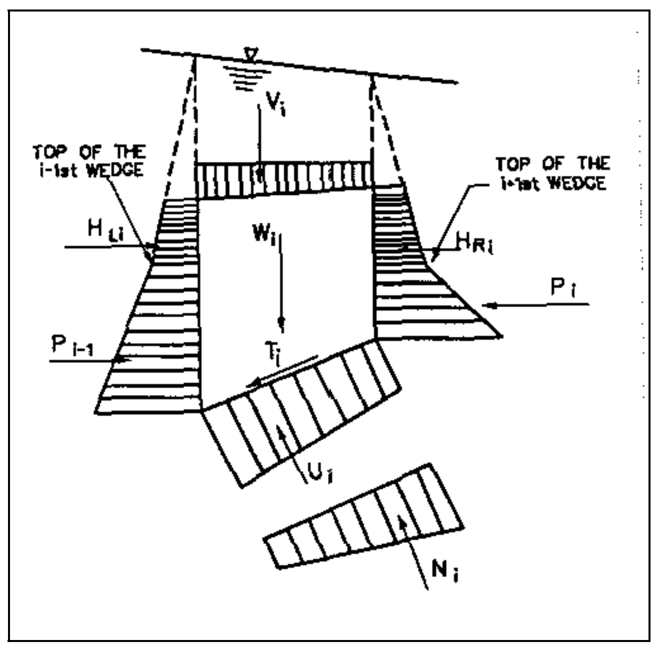


Figure C-3. Distribution of pressures and resultant forces acting on a typical wedge

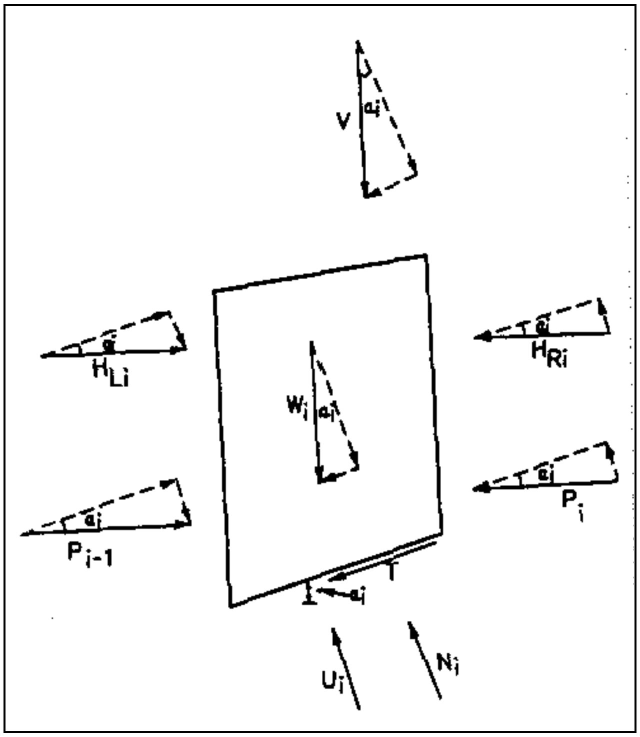


Figure C-4. Free body diagram of the i[th] wedge

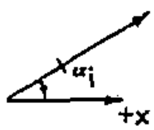
Equilibrium Equations

$$\Sigma F_n = 0$$

$$0 = N_i + U_i - W_i \cos \alpha_i - V_i \cos \alpha_i - H_{Li} \sin \alpha_i + H_{Ri} \sin \alpha_i + \cdots$$

$$\cdots - P_{i-1} \sin \alpha_i + P_i \sin \alpha_i$$

$$N_i = \left(W_i + V_i\right) \cos \alpha_i - U_i + \left(H_{Li} - H_{Ri}\right) \sin \alpha_i + \left(P_{i-1} - P_i\right) \sin \alpha_i$$



$$\Sigma F_s = 0$$

$$0 = -T_i - W_i \sin \alpha_i - V_i \sin \alpha_i + H_{Li} \cos \alpha_i - H_{Ri} \cos \alpha_i + \cdots$$

$$\cdots + P_{i-1} \cos \alpha_i - P_i \cos \alpha_i$$

$$T_i = \left(H_{Li} - H_{Ri}\right) \cos \alpha_i - \left(W_i + V_i\right) \sin \alpha_i + \left(P_{i-1} - P_i\right) \cos \alpha_i$$

Mohr-Coulomb Failure Criterion

$$T_F = N_i \tan \phi_i + c_i L_i$$

Safety Factor Definition

$$FS_i = \frac{T_F}{T_i} = \frac{N_i \tan \phi_i + c_i L_i}{T_i}$$

Figure C-5. Derivation of the general equation (Sheet 1 of 3)

Governing Wedge Equation

$$FS_i = \frac{\left\{\left(W_i + V_i\right)\cos\alpha_i - U_i + \left[\left(H_{Li} - H_{Ri}\right) + \left(P_{i-1} - P_i\right)\right]\sin\alpha_i\right\}\tan\phi_i + c_i L_i}{\left[\left(H_{Li} - H_{Ri}\right) + \left(P_{i-1} - P_i\right)\right]\cos\alpha_i - \left(W_i + V_i\right)\sin\alpha_i}$$

$$\left(P_{i-1} - P_i\right)\left(\cos\alpha_i - \sin\alpha_i \frac{\tan\phi i}{FS_i}\right) = \left[\left(W_i + V_i\right)\cos\alpha_i - U_i + \left(H_{Li} - H_{Ri}\right)\sin\alpha_i\right]\frac{\tan\phi_i}{FS_i} + \cdots$$

$$\cdots + \frac{c_i}{FS_i} L_i - \left(H_{Li} - H_{Ri}\right)\cos\alpha_i + \left(W_i + V_i\right)\sin\alpha_i$$

$$\left(P_{i-1} - P_i\right) = \frac{\left[\left(W_i + V_i\right)\cos\alpha_i - U_i + \left(H_{Li} - H_{Ri}\right)\sin\alpha_i\right]\frac{\tan\phi_i}{FS_i} - \left(H_{Li} - H_{Ri}\right)\cos\alpha_i + \left(W_i + V_i\right)\sin\alpha_i + \frac{c_i}{FS_i} L_i}{\left(\cos\alpha_i - \sin\alpha_i \frac{\tan\phi_i}{FS_i}\right)}$$

NOTE: A negative value of the difference (P_{i-1} - Pi) indicates that the applied forces acting on the i^{th} wedge exceed the forces resisting sliding along the base of the wedge. A positive value of the difference (P_{i-1} - Pi) indicates that the applied forces acting on the i^{th} wedge are less than the forces resisting sliding along the base of the wedge.

Figure C-5. (Sheet 2 of 3)

Solution for the Safety Factor

The governing equation for $(P_{i-1} - P_i)$ applies to the individual wedges

$$\left(P_{i-1} - P_i\right) = \frac{\left[\left(W_i + V_i\right)\cos\alpha_i - U_i + \left(H_{Li} - H_{Ri}\right)\sin\alpha_i\right]\dfrac{\tan\phi_i}{FS_i} - \left(H_{Li} - H_{Ri}\right)\cos\alpha_i + \left(W_i + V_i\right)\sin\alpha_i + \dfrac{c_i}{FS_i}L_i}{\left(\cos\alpha_i - \sin\alpha_i\dfrac{\tan\phi_i}{FS_i}\right)}$$

For the system of wedges to act as an integral failure mechanism, the safety factors for all wedges must be identical

$$FS_1 = FS_2 = \cdots = FS_{i-1} = FS_i = FS_{i+1} = \cdots FS_N$$

N = Number of wedges in the failure mechanism

The actual safety factor (FS) for sliding equilibrium is determined by satisfying overall horizontal equilibrium ($\Sigma F_H = 0$) for the entire system of wedges

$$\sum_{i=1}^{N}\left(P_{i-1} - P_i\right) = 0$$

And: $P_o = 0$ $P_N = 0$

Usually an iterative solution process is used to determine the actual safety factor for sliding equilibrium.

Figure C-5. (Sheet 3 of 3)

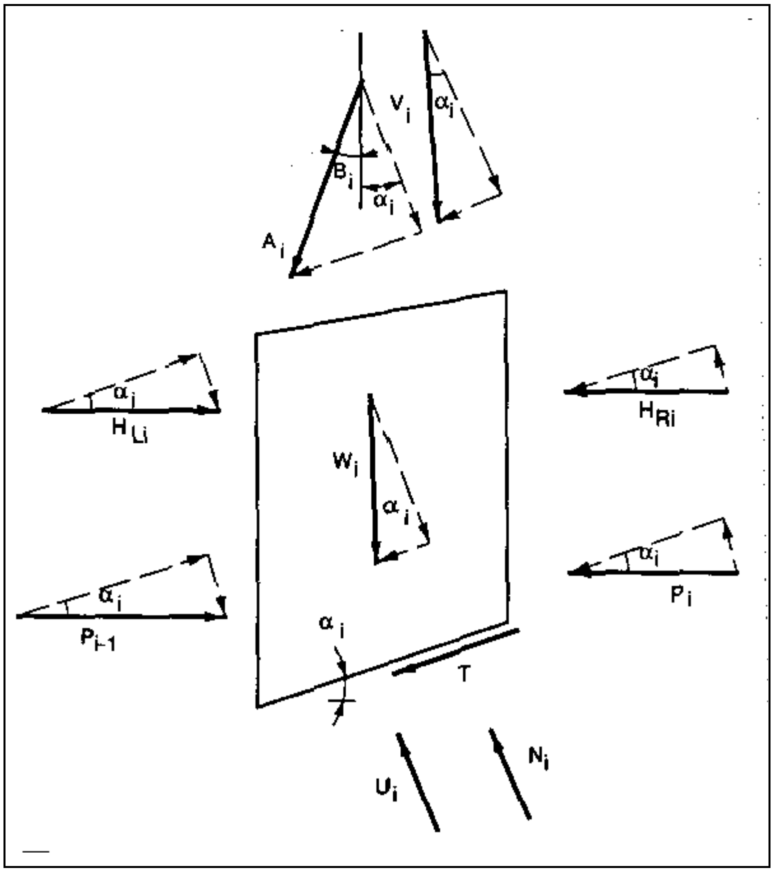
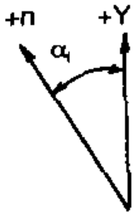


Figure C-6. Free body diagram of the i^{th} wedge with anchor

The free body diagram above varies from that shown in Figure C-4 in that the above free body diagram contains an anchor force "A_i" oriented at an angle, β_i with the vertical. The equilibrium equations and governing wedge equation on the following pages will include this anchor force.

Equilibrium Equations



$$\Sigma F_n = 0$$

$$0 = N_i + U_i - W_i \cos\alpha_i - V_i \cos\alpha_i - A_i \cos\left(\beta_i + \alpha_i\right) - H_{Li} \sin\alpha_i + H_{Ri} \sin\alpha_i - P_{i-1} \sin\alpha_i + P_i \sin\alpha_i$$

$$N_i = \left(W_i + V_i\right)\cos\alpha_i + A_i \cos\left(\beta_i + \alpha_i\right) - U_i + \left(H_{Li} - H_{Ri}\right)\sin\alpha_i + \left(P_{i-1} - P_i\right)\sin\alpha_i$$

$$\Sigma F_t = 0$$

$$0 = -T_i - W_i \sin\alpha_i - V_i \sin\alpha_i + H_{Li} \cos\alpha_i - H_{Ri} \cos\alpha_i + P_{i-1} \cos\alpha_i - P_i \cos\alpha_i - A_i \sin\left(\beta_i + \alpha_i\right)$$

$$T_i = \left(H_{Li} - H_{Ri}\right)\cos\alpha_i - \left(W_i + V_i\right)\sin\alpha_i + \left(P_{i-1} - P_i\right)\cos\alpha_i - A_i \sin\left(\beta_i + \alpha_i\right)$$

Mohr-Coulomb Failure Criterion

$$T_F = N_i \tan\phi_i + c_i L_i$$

Safety Factor Definition

$$FS_i = T_F \,/\, T_i = \left(N_i \tan\phi_i + c_i L_i\right) / \left(T_i\right)$$

Figure C-7. Derivation of the general equation for a wedge containing an anchor force (Sheet 1 of 3)

Governing Wedge Equation

$$FS_i = \frac{\left[(W_i + V_i)\cos\alpha_i + A_i\cos\left(\beta_i + \alpha_i\right) - U_i + \left((H_{Li} - H_{Ri}) + (P_{i-1} - P_i)\right)\sin\alpha_i\right]\tan\phi_i + c_i L_i}{\left[(H_{Li} - H_{Ri}) + (P_{i-1} - P_i)\right]\cos\alpha_i - (W_i + V_i)\sin\alpha_i - A_i\sin\left(\beta_i + \alpha_i\right)}$$

$$\left(P_{i-1} - P_i\right)\left(\cos\alpha_i - \sin\alpha_i \cdot \tan\phi i \;/\; FS_i\right) =$$

$$\left[(W_i + V_i)\cos\alpha_i + A_i\cos\left(\beta_i + \alpha_i\right) - U_i + (H_{Li} - H_{Ri})\sin\alpha_i\right]\tan\phi_i \;/\; FS_i \cdots$$

$$\cdots - (H_{Li} - H_{Ri})\cos\alpha_i + (W_i + V_i)\sin\alpha_i + A_i\sin\left(B_i + \alpha_i\right) + c_i L_i \;/\; FS_i$$

$$\left(P_{i-1} - P_i\right) = \frac{\left[(W_i + V_i)\cos\alpha_i + A_i\cos\left(\beta_i + \alpha_i\right) - U_i + (H_{Li} - H_{Ri})\sin\alpha_i\right]\tan\phi_i \;/\; FS_i}{\cos\alpha_i - \left(\sin\alpha_i\tan\phi_i \;/\; FS_i\right)} \cdots$$

$$\cdots + \frac{-(H_{Li} - H_{Ri})\cos\alpha_i + (W_i + V_i)\sin\alpha_i + A_i\sin\left(\beta_i + \alpha_i\right) + c_i L_i \;/\; FS_i}{\cos\alpha_i - \left(\sin\alpha_i\tan\phi_i \;/\; FS_i\right)}$$

NOTE: A negative value of the difference (P_{i-1} - P_i) indicates that the applied forces acting on the i^{th} wedge exceed the forces resisting sliding along the base of the wedge. A positive value of the difference (P_{i-1} - P_i) indicates that the applied forces acting on the i^{th} wedge are less than the forces resisting sliding along the base of the wedge.

Figure C-7. Derivation of the general equation for a wedge containing an anchor force (Sheet 2 of 3)

Solution for the Safety Factor

The governing equation for $(P_{i-1} - P_i)$ applies to the individual wedges

$$\left(P_{i-1} - P_i\right) = \frac{\left[(W_i + V_i)\cos\alpha_i + A_i\cos\left(\beta_i + \alpha_i\right) - U_i + (H_{Li} - H_{Ri})\sin\alpha_i\right]\tan\phi_i \,/\, FS_i}{\cos\alpha_i - \left(\sin\alpha_i\tan\phi_i \,/\, FS_i\right)} \ldots$$

$$\ldots + \frac{-(H_{Li} - H_{Ri})\cos\alpha_i + (W_i + V_i)\sin\alpha_i + A_i\sin\left(\beta_i + \alpha_i\right) + c_i L_i \,/\, FS_i}{\cos\alpha_i - \left(\sin\alpha_i\tan\phi_i \,/\, FS_i\right)}$$

Using the above governing equations for $(P_{i-1} - P_i)$ for wedges that may contain anchors, the solution for the safety factor is calculated as shown in page C-5.

Figure C-7. Derivation of the general equation for a wedge containing an anchor force (Sheet 3 of 3)

Note: Solutions of factors of safety and forces on free bodies determined by the above equations, or by computer analyses should be verified by an independent method of analysis. Vector diagrams may be used to check the results graphically. In multi-wedge analyses, consideration should be given to the stress-strain compatibility of different rock materials on the failure plane.